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# Power Computations for the Actor-Partner Interdependence Model

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# Power Computation for the Actor-Partner Interdependence Model

Presented here are methods to determine power in tests of actor and partner effects for the Actor-Partner Interdependence Model (APIM). Only the simple two-variable model, one mixed *X* variable which is presumed to determine on mixed *Y* variable. First, the indistinguishable case is discussed and then the distinguishable case.

## Indistinguishable Case

### Model

The basic model for the APIM

$$Y_1 = aX_1 + pX_2 + E_1$$

$$Y_2 = pX_2 + aX_1 + E_2$$

where *a* is the actor effect and *p* the partner effect. When *X* and *Y* are standardized variables, *a* and *p* become betas. We denote $r_{xx}$ as the actor-partner correlation and $r_{ee}$ is the correlation between the errors.

### Computation of Effect Sizes

For this model, there are three possibilities for effect size *a* and *b*: beta or $\beta$, partial *r* or $r_P$, and Cohen's *d*. The app APIMPower uses $\beta$ as the measure of the effect for all power computations. If either $r_P$ or *d* is chosen for a measure of effect size, each is converted into a $\beta$. Considering *d* first, where *P* is the proportion of persons in one category. The *X* variable is a dichotomy whose standard deviation equals:

$$s_X = \sqrt{\frac{NP(1-P)}{N-1}}.$$

If *N* is not known which happens when only desired power is sought, then we simply use the square root of $P(1 - P)$ as the standard deviation. For *d* the error variance is fixed to 1. That makes the variance for *Y* equal to

$s_X^2(d_a^2 + d_p^2 + 2d_ad_pr_{X1X2}) + 1$, where $d_a$ is the $d$ for the actor effect and $d_p$ is the $d$ for the partner effect and $s_X$ is defined as above. Note that each of the $d$s is a "partial" $d$, in that the other effect is controlled, i.e., the mean difference in $Y$ controlling for the other effect divided by the error variance. To turn the $d$'s into betas, we multiply each $d$ by $s_X/s_Y$, where $s_X$ and $s_Y$ are the pooled standard deviations of $X$ and $Y$, respectively.

We know of no closed-form formula to convert $r_P$ into $\beta$, and so that conversion must be done iteratively. We initially set the betas to be equal to the partial correlations. We then repeat the following four steps:

First, using the "betas" and the user-inputted correlation between the $X$s, the correlations with $Y$ which are denoted as $r_{X1Y}'$ and $r_{X2Y}'$, are estimated.

Second, using $r_{X2Y}'$ and $r_{X1Y.X2}$, we can solve for $r_{X1Y}''$.

Third, using $r_{X1Y}''$ and $r_{X1Y.X2}$, we can solve for $r_{X1Y}'''$.

Fourth, with $r_{X1Y}'''$ and $r_{X1Y}''$, we obtain new betas.

We then repeat these four steps to get an improved estimate of the betas. We do this until the estimated betas change by no more than 0.0000000001. Normally this takes no more than 12 steps. Because of the iterative computation of the betas from the partials, this analysis within APIMPower is slower than it is with the other measures of effect size.

**Power Computation**

To compute power, we need to know the standard error of $\beta$. We use the pooled-regression method described on pages 152-156 in Kenny et al. (2006). The method involves computation of a sum and difference of the $X$ and $Y$ variables and the sum $Y$ ($Y_1 + Y_2$) is regressed on sum $X$ ($X_1 + X_2$) and difference $Y$ ($Y_2 - Y_1$) is regressed on difference $X$ ($X_2 - X_1$) with no intercept. The squared standard error of the sum regression analysis or $s_S^2$ can be shown to equal

$$\frac{(1 - r_{ss}^2)(1 + r_{yy})}{(1 + r_{xx})(N - 2)}$$

where $r_{ss}$, the correlation of sum $X$ with sum $Y$, can be shown to equal

$$(a + p)\sqrt{\frac{1\ +\ r_{xx}}{1\ +\ r_{yy}}}$$

where $r_{yy}$ or the correlation of $Y$ across the two members can be shown to equal

$$2ap + r_{xx}\ (a^2 +\ p^2) +\ r_{ee}(1 - a^2\ - b^2 - 2apr_{xx}).$$

The squared standard error of the regression of differences or $s_D^2$ can be shown to equal

$$\frac{(1 - r_{dd}^{\ 2})(1 -\ r_{yy})}{(1\ -\ r_{xx})(N - 1)}$$

where $r_{dd}$, the correlation of difference $X$ with difference, can be shown to equal

$$(a - p)\sqrt{\frac{1 -\ r_{xx}}{1 -\ r_{yy}}}$$

where $r_{yy}$ is defined as above.   These two squared standard errors, $s_D^2$ and $s_P^2$, are added together, divided by four, and that quantity is square rooted or $\sqrt{[(s_D^2 + s_S^2)/4]}$ to yield the standard error for both $a$ and $p$.

The degrees of freedom of the pooled standard error are given by the Satterthwaite method and equal

$$\frac{({s_S}^2\ + {s_D}^2)^2}{\frac{{s_S}^4}{N - 2} + \frac{{s_D}^4}{N - 1}}$$

To obtain the non-centrality parameter, the regression coefficient is divided by its standard error.  With the non-centrality parameter and the Satterthwaite degrees of freedom, we can use the non-central $t$ distribution to compute the percentage of time the critical value for a given alpha is exceeded.

If there are singles, we can compute a usual standard error, $s_s^2$, for a regression coefficient which in this case would equal

$$\frac{1 - a^2 - p^2 - 2apr_{xx}}{(N_s - 2)(1 - r_{xx}^2)}$$

where $N_s$ is the number of singles. The degrees of freedom would be $N_s - 3$. Alternatively, for singles and the *distinguishable* case, one can convert the beta to a partial correlation or $r_p$ and uses the following formula for the noncentrality parameter

$$r_p \sqrt{\frac{N - 3}{1 - r_p^2}} \; .$$

We pool the standard errors and degrees of freedom for dyads and singles by weighting by the inverse of their standard errors squared. The formulas are as follows where $s_{DD}$ and $df_D$ are the standard error and degrees of freedom for dyads and $s_{SS}$ and $df_S$ for singles:

$w_1 = s_{DD}^2/(s_{DD}^2 + s_{SS}^2)$

$w_2 = s_{SS}^2/(s_{DD}^2 + s_{SS}^2)$

$df_P = (w_1^2 s_{DD}^2 + w_2^2 s_{DS}^2)^2/(w_1^4 s_{DD}^4/df_D + w_2^4 s_{DS}^4/df_S)$

$s_P^2 = w_1^2 s_{DD}^2 + w_2^2 s_{DS}^2$ .

We compute $a/s_P$ and $p/s_P$ to obtain the non-centrality parameters for actor and partner effects, respectively.

## Distinguishable Case

### Model

The basic model is

$$Y_1 = a_1X_1 + p_1X_2 + E_1$$

$$Y_2 = p_2X_1 + a_2X_2 + E_2$$

The two actor and partner effects are denoted by the $Y$ variable.

### Computation of Effect Size

In APIMPower and the distinguishable case, effect sizes need to be entered for both persons 1 and 2. The relative variance of $X$ and the relative variance of the errors in $Y$ for the two members might vary across the two members. APIMPower always fixes the variance of $X_1$ and $Y_1$ to one, which always makes a standardized value, i.e., a beta. However, $p_1$ would be standardized only if the variances of $X_1$ and $X_2$ were equal. Almost never would $a_2$ and $p_2$ be standardized values

To understand some of the complexity here, consider the case in which the two actor effects equal 0.5 and the partner effect for member 1 equals 0.3 and for member 2 equals zero:

$$Y_1 = 0.5X_1 + 0.3X_2 + E_1$$

$$Y_2 = 0.5X_2 + 0.0X_1 + E_2$$

We set $r_{X1X2} = 0$ and that both $X_1$ and $X_2$ have variances of one. If we fix the variance of $Y_1$ to one, then the variance of $E_1$ equals 0.66 (i.e., $1 – 0.5^2 – 0.3^2$). If we fix the variances of the errors to be equal across members, we note then that the variance of $Y_2$ is not one, but 0.91 (0.25 + 0.66). Thus, although the coefficients for $Y_1$ are beta weights (i.e., standardized), those for $Y_2$ are not.

The formulas for the variances are as follows. We fix the variance in $Y_1$ and $X_1$ to equal 1. The variance of $X_2$ to equal to $q^2$ which makes $q$ equal to $s_{X2}/s_{X1}$. If we solve for the variance of $E_1$ we obtain: $1 – a_1^2 – q^2p_1^2 – 2q^2a_1p_1r_{12}$. We denote $w$ as the ratio of $s_{E2}/s_{E1}$, which implies that $s_{E2}^2 = w^2s_{E1}^2$. The variance of $Y_2$ can then be shown to equal

$$q^2a_2^2 + p_2^2 + 2q^2a_1p_1r_{12} + w^2(1 – a_1^2 – q^2p_1^2 – 2qa_1p_1r_{12})$$

Consider the case in which the error variance for person 2 is twice as great as the error variance for person 1 ($w^2 = 2$), but the variance of $X$ for person 2 is half the size of the variance of $X$ for person 2, $q^2 = .5$. We also set actor and partner effects equal, $a_1 = a_2$ and $p_1 = p_2$. In this case although the actor effect from person 2 is the same value as it is for person 1, if follows that $\beta_{A2}$ is half as large as $\beta_{A1}$. Note for the partner effect, the actor and partner variances flip making $\beta_{P2} = \beta_{P1}(s_{X1}/s_{X2})(s_{E1}/s_{E2})$. Note then if actor and partner effects are the same for both members, but say person 2 has more variance than person 1, it would be much more difficult to find partner effects for Person 2.

When $d$ is the measure of effect size, we need to also know the percentage of persons in each category. We denote $P_1$ as the proportion in the category for $X_1$ and $P_2$ for $X_2$. To compute the betas, we need to know the variances of $X$ and $Y$. The variance of $X_1$ equals $N(P_1(1 – P_1)/(N - 1)$, the variance of $X_2$ equals $N(P_2(1 – P_2)/(N - 1)$, the variance of $Y_1$ equals

$$s_{X1}{}^2 d_{a1}{}^2 + s_{X2}{}^2 d_{p1}{}^2 + 2d_{a1}d_{p1}s_{X1}s_{X2}r_{X1X2} + 1$$

where $d_{a1}$ is the actor $d$ for person 1 and $d_{p1}$ is the partner $d$ for person 1 (effect of person 2 on person 1), and the variance of $Y_2$ equals

$$s_{X2}{}^2 d_{a2}{}^2 + s_{X1}{}^2 d_{p2}{}^2 + 2d_{a2}d_{p2}s_{X1}s_{X2}r_{X1X2} + 1$$

where $d_{a2}$ is the actor $d$ for person 2 and $d_{p2}$ is the partner $d$ for person 2. Note that if the user specified that the error variances differ, then the variance of $Y_2$ needs to be multiplied by that factor. Thus, if say that $d_{a1} = d_{p1} = 0.30$, but there is 4 times more error variance for $Y_2$ than $Y_1$, for the effects to be equal we need to divide the $d$s for member two by the square root of 4 and so they would both equal 0.15.

To compute effect sizes when the effect size is the partial correlation, we first convert the partials to betas as was described earlier for the indistinguishable case. Then we can use $\beta_{X1Y1}$ for $a_1$ and $\beta_{X2Y1}/q$ (where again $q$ equals $s_{X2}/s_{X1}$) for $p_1$. It can be shown

$$s_{Y2}{}^2 = w^2(1 – a_1{}^2 – q^2p_1{}^2 – 2qa_1p_1r_{12})/(1 - r_{X1Y2}\beta_{X1Y2} - r_{X1Y2}\beta_{X1Y2})$$

Then $a_2$ equals $s_{Y2}\beta_{X2Y2}/q$ for $p_1$ equals $s_{Y2}\beta_{X1Y2}$.

**Power Computation**

To compute power, we need to determine the standard errors of $a_1$, $a_2$, $p_1$, and $p_2$. Those squared standard errors are for $a_1$

$$\frac{s_{Y_1}(1 - a_1{}^2 - p_1{}^2 - 2a_1p_1r_{xx})}{s_{X_1}(N-2)(1 - r_{xx}^2)},$$

for $p_1$

$$\frac{s_{Y_1}(1 - a_1{}^2 - p_1{}^2 - 2a_1p_1r_{xx})}{s_{X_2}(N-2)(1 - r_{xx}^2)},$$

for $a_2$

$$\frac{s_{Y_2}(1 - {a_2}^2 - {p_2}^2 - 2a_2p_2r_{xx})}{s_{X_2}(N-2)(1 - \ r_{xx}^2)},$$

and for $p_1$

$$\frac{s_{Y_2}(1 - {a_2}^2 - {p_2}^2 - 2a_2p_2r_{xx})}{s_{X_2}(N-2)(1 - \ r_{xx}^2)},$$

with degrees of freedom of $N - 3$. The key then is the computation of the standard deviations for $X_1$, $X_2$, $Y_1$, and $Y_2$. As stated above, the variances of $X_1$ and $Y_1$ are fixed to one, the variance of $X_2$ is defined as $q^2$, and the variance of $Y_2$ is defined as

$$q^2a_2^2 + p_2^2 + 2q^2a_1p_1r_{12} + w^2(1 - a_1^2 - q^2p_1^2 - 2q^2a_1p_1r_{12})$$

where again $w$ is defined as $s_{E2}/s_{E1}$. Knowing these variances, the standard errors can be computed and the non-centrality parameter can be computed, making the computation of power straightforward.

Singles occur when both $X_1$ and $X_2$ for the both persons, but $Y$ is measured on just one person. Note that for the distinguishable case there can be a different number of singles for each member. The only affect singles have on the computation of the non-centrality parameters and degrees of freedom is the value for $N$ which is set equal to the number of dyads plus the number of singles.

**Difference in Actor and Partner Effects**

Researchers often test if actor or partner effects are equal to one another: $a_1 = a_2$ and $p_1 = p_2$. The variance in difference between actor effects is $s_{a1}^2 + s_{a2}^2 + 2r_{xx}r_{ee}s_{a1}s_{a2}$ given that the correlation of $a_1$ and $a_2$. equals $-r_{xx}r_{ee}$. The variance in difference between partner effects equals $s_{p1}^2 + s_{p2}^2 + 2r_{pp}s_{p1}s_{p2}$. Currently, we do not know how to calculate the degrees of freedom of either of these. However, with the standard errors, we can calculate power assuming a $Z$ test. Note that the standard error for $(a_1 + a_2)/2 = 0$ and $(p_1 + p_2)/2 = 0$ are $(s_{a1}^2 + s_{a2}^2 - 2r_{xx}r_{ee}s_{a1}s_{a1})/4$ and $(s_{p1}^2 + s_{p2}^2 - 2r_{xx}r_{ee}s_{p1}s_{p1})/4$, respectively.