Estimating the Variances and Covariances for the Reciprocal One-with-Many Design

Deborah A. Kashy **David A. Kenny**

Michigan State University **University of Connecticut**

There are two records for each dyad, one for focal person and one for the partner. The following variables must be created:

Focalid: an identification number that specifies the focal person. In the example, Focalid ranges from 1 to 21 because the total number of focal persons in the data set is 21

Dyadid: an identification number that specifies the dyad. In the example, Dyadid ranges from 1 to 101 because the total number of partners is 101.

Focalcode: 1 if the observed rating was made by the focal person and 0 otherwise

Partcode: 1 if the observed rating was made by the partner and 0 otherwise

Outcome: the variable is measured from both the focal person and the partners

Role: a variable that specifies who the rater is; for example, -1 = partner, 1 = focal person or "Focal" and "Part"

Focalid	Dyadid	Focalcode	Partcode	Outcome	Role
1	1	0	1	6.00	-1
1	1	1	0	8.25	1
1	2	0	1	8.75	-1
1	2	1	0	6.00	1
1	3	0	1	8.00	-1
1	3	1	0	8.00	1
1	4	0	1	8.25	-1
1	4	1	0	8.00	1
2	5	0	1	5.00	-1
2	5	1	0	8.75	1
2	6	0	1	6.50	-1
2	6	1	0	7.75	1
2	7	0	1	5.75	-1
2	7	1	0	8.75	1

SPSS Syntax to estimate the variance partitioning and covariances

```
MIXED
Outcome  BY role  WITH focalcode partcode
/FIXED = focalcode partcode | NOINT
/PRINT = SOLUTION TESTCOV
/RANDOM focalcode partcode | SUBJECT(focalid) COVTYPE(UNR)
/REPEATED = role | SUBJECT(focalid*dyadid) COVTYPE(UNR).
```

(The bold terms are required SPSS terms; the unbolded terms are variable names.) The two dummy codes for focal person (focalcode) and partner (partcode) serve to estimate a two intercept model in which separate intercepts are estimated for focal persons and partners. Note that the traditional intercept is suppressed (i.e., NOINT).

The Random statement includes the two dummy codes for focal person and partner. This results in estimates in the variance in the intercepts. For focal person this variance estimates actor variance and for the partner this variance estimates the partner variance. Because COVTYPE is UN (unspecified), this also estimates a covariance which is the Actor-Partner covariance. Alternatively, one could state UNR to obtain a correlation.

The repeated statement is necessary to specify the unique variances and the dyadic covariance. It specifies that role is repeated within the focalid-dyadid combinations. Because COVTYPE is UN (unspecified), this also estimates a covariance which is the dyadic reciprocity covariance. Alternatively, one could state UNR to obtain a correlation.

By Point-and-Click

Step 1: Start
- ANALYZE
- MIXED MODELS
- LINEAR
- Move in FOCALID and DYADID in SUBJECTS
- Type ROLE in REPEATED
- On the bottom, change DIAGONAL to UNSTRUCTURED: CORRELATION METRIC
- CONTINUE

Step 2: Linear Mixed Models
- Type in the name of the DEPENDENT VARIABLE
- Type categorical variables in FACTOR(S)
  - Make ROLE a factor.
- Type continuous variables in COVARIATE(S)
  - Make PARTCODE and PARTCODE covariates.
  - Add in any other predictors.

— The remaining steps go from left to right on the bottom of the screen. —

Step 3: Fixed
- Add in PARTCODE and PARTCODE as main effects.
- Add in any other relevant terms.
- Make sure “INCLUDE INTERCEPT” box is **NOT** checked.
- CONTINUE

Step 4: Random

On the bottom of "SUBJECT GROUPINGS," move FOCALID from SUBJECTS into COMBINATIONS.

Do **NOT** click "INCLUDE INTERCEPT"

In the upper screen move over PARTCODE and PARTCODE to the right side as main effects.

In the top of the screen, change VARIANCE COMPONENTS to UNSTRUCTURED:

CORRELATION METRIC.

CONTINUE

Step 5: Statistics

Click PARAMETER ESTIMATES

Click TESTS FOR COVARIANCE PARAMETERS

CONTINUE

You may want to click PASTE to save syntax.

Step 6: Run the job

Click OK

As an example we use the Cook data presented in Chapter 10.

Mixed Model Analysis – Partial output

Fixed Effects

						95% Confidence Interval	
Parameter	Estimate	Std. Error	df	t	Sig.	Lower Bound	Upper Bound
focalcode	1.698269	.034249	207.000	49.587	.000	1.630748	1.765790
partcode	1.807695	.040989	207.000	44.102	.000	1.726886	1.888505

Dependent Variable: outcome.

The estimates above show the intercept which is close to the mean of the ratings made by the mother (Focalcode estimate is 1.698). The partcode estimate indicates the average outcome score across partners of the mother which is larger than mothers' anxiety.

Covariance Parameters

						95% Confidence Interval	
Parameter		Estimate	Std. Error	Wald Z	Sig.	Lower Bound	Upper Bound
Repeated Measures	Var(1)	.4231554	.0293406	14.422	.000	.3693854	.4847526
	Var(2)	.5492344	.0380826	14.422	.000	.4794436	.6291844
	Corr(2,1)	.2390289	.0462278	5.171	.000	.1465852	.3273337
focalcode + partcode [subject = focalid]	Var(1)	.0608985	.0271341	2.244	.025	.0254298	.1458378
	Var(2)	.2084094	.0357153	5.835	.000	.1489519	.2916008
	Corr(2,1)	.6988175	.1709956	4.087	.000	.2069308	.9086992

a Dependent Variable: outcome.

These results are identical to those presented in Table 10.4, with some rounding error.

Reading from top down:

The repeated measures UN(1,1) provides the relationship variance for Role = 1 (the smaller role value or the Role value early in the alphabet) or mothers' judgments of their anxiety with family members. Thus, the relationship variance for the partners is .423. The repeated measures UN(2,2) provides the relationship variance for Role = 1 and equals 1.249. The repeated measures UN(2,1) provides the covariance between the two relationship effects. Mothers who are particularly anxious with one family member elicit anxiety from that family member.

The next three estimates are for the variances and covariances of the individual-level effects. Because Focalcode was the first variable in the RANDOM statement, UN(1,1) is the actor variance.. With SPSS, the *p* values for variances should be divided two. Thus, for the partner variance the value is .0125. UN(2,1) is the actor-partner covariance, and UN(2,2) is the partner variance.

Covariates

If there are covariates, there are two options. Note that there are in essence two outcome variable, one for the focal persons and one for the partners. To set the effect of the covariate be the same for both variables, the covariate would be included in the fixed statement. If, however, the researcher wanted to allow the effects of the covariate to be different for the focal person and the partners, then the covariate would be entered as covariate*role in the fixed statement. This would result in two regression coefficients, one for each of the two persons.

SAS

Syntax

```
Proc MIXED cl covtest;
  class focalid dyadid;
 model outcome =  focalcode partcode  / NOINT  S DDFM=SATTERTH;
   RANDOM  focalcode partcode   / SUBJECT=focalid TYPE=UN;
  REPEATED  / SUBJECT= focalid*dyadid TYPE=UN ;
run;quit;
```

Partial Output

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	focalid	0.06090	0.02713	2.24	0.0124
UN(2,1)	focalid	0.07873	0.02332	3.38	0.0007
UN(2,2)	focalid	0.2084	0.03572	5.84	<.0001
UN(1,1)	focalid*dyadid	0.4232	0.02934	14.42	<.0001
UN(2,1)	focalid*dyadid	0.1152	0.02430	4.74	<.0001
UN(2,2)	focalid*dyadid	0.5492	0.03808	14.42	<.0001

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
focalcode	1.6983	0.03425	207	49.59	<.0001
partcode	1.8077	0.04099	207	44.10	<.0001

HLM

Output

```
D
 FOCALCOD,P1      0.54923      0.11523
 PARTCODE,P2      0.11523      0.42316


Standard Errors of D
 FOCALCOD,P1      0.03808      0.02430
 PARTCODE,P2      0.02430      0.02934


D (as correlations)
 FOCALCOD,P1  1.000  0.239
 PARTCODE,P2  0.239  1.000

Tau(beta)
 FOCALCOD     PARTCODE
 INTRCPT2,B10 INTRCPT2,B20
    0.05973      0.07816
    0.07816      0.20673

Standard Errors of Tau(beta)
 FOCALCOD     PARTCODE
 INTRCPT2,B10 INTRCPT2,B20
    0.02698      0.02317
    0.02317      0.03548

Tau(beta) (as correlations)
 FOCALCOD/INTRCPT2,B10  1.000  0.703
 PARTCODE/INTRCPT2,B20  0.703  1.000

 The outcome variable is  OUTCOME

 Final estimation of fixed effects:
 ----------------------------------------------------------------------------
                                  Standard             Approx.
   Fixed Effect        Coefficient Error      T-ratio   d.f.     P-value
 ----------------------------------------------------------------------------
 For FOCALCOD slope, P1
    For INTRCPT2, B10
      INTRCPT3, G100       1.698269   0.034166    49.706       207    0.000
 For PARTCODE slope, P2
    For INTRCPT2, B20
      INTRCPT3, G200       1.807695   0.040890    44.208       207    0.000
```

MLwiN

$$\text{Outcome}_{ijk} \sim \text{N}(XB,\ \Omega)$$

$$\text{Outcome}_{ijk} = \beta_{0jk}\text{FocalCode}_{ijk} + \beta_{1jk}\text{DyadCode}_{ijk}$$

$$\beta_{0jk} = 1.699(0.034) + v_{0k} + u_{0jk}$$

$$\beta_{1jk} = 1.808(0.041) + v_{1k} + u_{1jk}$$

$$\begin{bmatrix} v_{0k} \\ v_{1k} \end{bmatrix} \sim \text{N}(0,\ \Omega_v) \ :\ \Omega_v = \begin{bmatrix} 0.061(0.027) & \\ 0.079(0.023) & 0.208(0.036) \end{bmatrix}$$

$$\begin{bmatrix} u_{0jk} \\ u_{1jk} \end{bmatrix} \sim \text{N}(0,\ \Omega_u) \ :\ \Omega_u = \begin{bmatrix} 0.549(0.038) & \\ 0.115(0.024) & 0.423(0.029) \end{bmatrix}$$

*-2*loglikelihood(IGLS Deviance)* = 2816.103(1248 of 1248 cases in use)