

Technical Appendix:
APIM Moderation Paper

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Indistinguishable Dyads and No Moderators

In this case there are no moderators, but we still discuss it to lay a basis for the case where there are moderator effects.

Multilevel Modeling (MLM)

For MLM, a pairwise dataset is created and a single equation is estimated for both dyad members with own X and partner's X as predictors. Nonindependence is modeled either as a random intercept at the level of the dyad or the correlation of errors. When using MLM, a pairwise dataset is created. That dataset has the variables of the outcome (Y), the actor variable (X), and the partner variable (X'). The model for person i in dyad j is

$$Y_{ij} = b_0 + b_1X_{ij} + b_2X'_{ij} + e_{ij} \quad (1)$$

where errors are correlated across members. The SPSS syntax for this model is

```
MIXED Y_A WITH X_A X_P
  /FIXED= X_A X_P
  /PRINT=SOLUTION TESTCOV
  /METHOD = ML
  /REPEATED=PARTNUM | SUBJECT(DYADID) COVTYPE(CSR) .
```

where PARTNUM is a variable that equals 1 for member 1 and 2 for member 2. For MLM in this document, we use maximum likelihood (ML) estimation instead of the more conventional restricted maximum likelihood (REML) in order to make estimates comparable with Structural Equation Modeling. Also ML would be required if submodels were compared that had different fixed effects.

Using the first example dataset from the paper, depression as a predictor variable and happiness with role responsibilities as the outcome we estimated the basic APIM treating dyads as indistinguishable and thus ignoring gender. The syntax was as follows:

```
MIXED RESPAR_A WITH DEPScore_A DEPScore_P
  /FIXED= DEPScore_A DEPScore_P
  /PRINT=SOLUTION TESTCOV
  /METHOD = ML
  /REPEATED= PARTNUM | SUBJECT(FAMID) COVTYPE(CSR) .
```

Both actor's depression and partner's depression were entered as predictors of happiness in the model. We find the following estimates: $b_0 = 4.324, p < .01, b_1 = -0.025, p < .01, b_2 = -0.027, p < .01$. Thus, there are both statistically significant actor and partner effects of depression on happiness with role responsibility and these effects are both negative and comparable in magnitude.

Structural Equation Modeling

When using SEM, we adopt the Olsen and Kenny (2006) approach involving equality constraints. We have a dyad dataset, and two equations

$$Y_1 = b_0 + b_1X_1 + b_2X_1 + e_1 \quad (2)$$

$$Y_2 = b_0 + b_2X_1 + b_1X_1 + e_2 \quad (3)$$

Note that the intercepts as well as the actor and partner effects are constrained to be equal across equation. Additionally, we also constrain $V(e_1) = V(e_2)$, $V(X_1) = V(X_2)$, and the mean of X_1 and X_2 are fixed to be equal. The resulting model has 6 degrees of freedom, but it is treated as the saturated model and is called the I-SAT (indistinguishable and saturated) model by Olsen and Kenny (2006).

Using the example data from the paper, we estimated this basic APIM indistinguishable dyads model in AMOS 18. Figure 1 contains the AMOS model and the constraints imposed on this model to account for the indistinguishability of the dyads. The estimates of intercept, actor, and partner effects are identical to those that were found for MLM. We find the following estimates: $b_0 = 4.324, p < .01, b_1 = -0.025, p < .01, b_2 = -0.027, p < .01$. We find that this model that treats the dyads as indistinguishable and includes no moderators was a poor fit to the data $\chi^2(6) = 33.096, p < .01, CFI = .581, RMSEA = .125$. Because the dyads are truly distinguishable by gender, this result indicates that gender is important in this model. If, however, the dyads were truly indistinguishable, this chi square would not be information as it would reflect only the nonrandom assignment of members to the roles of "1" and "2."

Distinguishable Dyads (Dichotomous Within-Dyads Moderator)

Multilevel Modeling

When using MLM, the interaction of the distinguishing variable with the actor and partner effects tests moderating effects. The model for person i in dyad j with gender as the distinguishing variable, denoted as G , that acts as moderator is

$$Y_{ij} = b_0 + b_1X_{ij} + b_2X'_{ij} + b_3G_{ij} + b_4X_{ij}G_{ij} + b_5X'_{ij}G_{ij} + e_{ij}, \quad (4)$$

where X is the actor variable, X' is the partner variable, b_0 is the intercept, b_1 is the coefficient of X_{ij} on Y_{ij} (actor effect), b_2 is the coefficient of X'_{ij} on Y_{ij} (partner effect), b_4 is the actor interaction with gender, b_5 is the partner interaction with gender, and e_{ij} represents the residual term. If the interaction of gender and the actor variable is statistically significant, then the actor effect for husbands is statistically different from the actor effect for wives. That is, gender moderates the actor effect. Likewise, the interaction of gender and the partner variable indicates if the two partner effects statistically differ. To increase interpretability of the difference between actor and partner effects for the two members and test whether each member's actor and partner effects are statistically different from zero, a "two-intercept" model (Kenny et al., 2006; Raudenbush, Brennan, & Barnett, 1995) can be estimated, which is

$$Y_{ij} = b_1W_i + b_2H_i + b_3X_{ij}W_i + b_4X'_{ij}W_i + b_5X_{ij}H_i + b_6X'_{ij}H_i + E_{ij} \quad (5)$$

where W and H are two dummy coded variables for wife and husband (Kenny & Kashy, 2011). Such an approach estimates the actor and partner effects and intercepts for each dyad member. In sum, the interaction approach tests whether the actor and partner effects differ across levels of the within-dyads variable, and the two-intercept approach estimates each member's actor and partner effects and tests if they are statistically different from zero. We note that the interaction approach and the two-intercept approach are statistically identical but allow for different conclusions.

For the first example dataset used in the paper and in the indistinguishable examples above, we estimated the interaction approach and two-intercept models. For the interaction approach model the syntax was as follows:

```
MIXED RESPAR WITH DEPScorec_A DEPScorec_P GENDER
  /FIXED= DEPScorec_A DEPScorec_P GENDER DEPScorec_P*GENDER
          DEPScorec*GENDER
  /PRINT=SOLUTION TESTCOV
  /METHOD = ML
  /REPEATED= GENDER | SUBJECT(FAMID) COVTYPE(CSH) .
```

Gender was effect coded with women as 1 and men as -1. Depression scores are centered in these analyses. We find the following estimates: $b_0 = 3.905, p < .01$, $b_1 = -0.026, p < .01$, $b_2 = -0.026, p < .01$, $b_3 = -0.150, p < .01$, $b_4 = 0.004, p = .460$, $b_5 = -0.011, p = .074$. There are statistically significant main effects actor depression and partner depression on happiness. In addition, these effects are not different between men and women—gender does not moderate the actor and partner effects—although the interaction term is marginal for the partner effect indicating that the husbands’ depression has a bigger effect on the wives’ happiness than the wives’ depression has on the husbands’ happiness. There is also a statistically significant main effect of gender such that men are happier with their role responsibilities than women.

With the two-intercept approach, we can estimate the actor and partner effects for men and women separately. Using the same data and variables we estimated this model with the following syntax:

```
MIXED RESPAR BY GENDER WITH DEPScorec_A DEPScorec_P
  /FIXED= GENDER DEPScorec_A*GENDER DEPScorec_P*GENDER|NOINT
  /PRINT=SOLUTION TESTCOV
  /METHOD = ML
  /REPEATED=GENDER | SUBJECT(FAMID) COVTYPE(CSH) .
```

The estimates of the effects from Equation 5 were as follows: $b_1 = 3.755, p < .01$, $b_2 = 4.055, p < .01$, $b_3 = -0.021, p = .013$, $b_4 = -0.037, p < .01$, $b_5 = -0.030, p < .01$, $b_6 = -0.015, p = .082$. The partner effect for men (b_4) is not significantly different from zero while the partner effect for women (b_6) is

significantly different from zero. We might be tempted to conclude that the partner effect is moderated by gender. However, recall from the interaction approach model that these two partner effects were not statistically different from each other ($p = .074$). This example illustrates why estimating actor and partner effects separately for men and women is inadequate.

Structural Equation Modeling

For SEM, a dyad dataset is created and members are denoted as person 1 and 2. The APIM is defined by two structural equations, one for Y_1 and one for Y_2 . Both X_1 and X_2 predict Y_1 and Y_2 , the paths from X_1 to Y_1 and from X_2 to Y_2 being actor effects and the paths from X_2 to Y_1 and from X_1 to Y_2 being partner effects. For SEM, the distinguishable case is estimated by creating separate equations for each of the two members, which is for member 1

$$Y_1 = b_0 + b_1X_1 + b_2X_2 + e_1 \quad (6)$$

and for member 2 is

$$Y_2 = b_3 + b_4X_1 + b_5X_2 + e_2 \quad (7)$$

This model is saturated and so has zero degrees of freedom.

To test statistically if paths are equal across dyad members the two actor or two partner paths are set equal to each other and that equality constraint is evaluated by a chi-square test. The significance tests for the paths themselves evaluate whether the actor and partner effects are different from zero for each member. The main effect of the distinguishing variable is estimated by allowing the intercepts of Y_1 and Y_2 to differ.

For the example data we ran this model with wives as member 1 and husbands as member 2. The SEM model that needs to be estimates is the same as in Figure 1. The estimates from this model are the same as those from the two-intercept model within the MLM approach. The estimates are $b_0 = 3.755, p < .01, b_1 = -0.021, p = .013, b_2 = -0.037, p < .01$, for wives and $b_3 = 4.055, p < .01, b_4$

= -0.030, $p < .01$, $b_5 = -0.015$, $p = .082$, for husbands. There is no statistically significant partner effect for men.

To test if the actor and partner effects are statistically significantly different across member two additional models need to be estimated: 1) a model that fixes the two actor effects to be equal, and 2) another model that fixes the two partner effects to be equal, and then chi-square difference tests can be computed to assess whether or not there is a significant decline in fit when making these constraints. There is no significant decline in fit when constraining the actor effects to be equal, $\chi^2(1) = 0.544$, $p = .461$, and thus we can conclude that the actor effects are not different between husbands and wives. Similarly there is no significant decline in fit when constraining the partner effects, $\chi^2(1) = 3.190$, $p = .074$, indicating that the partner effects are not different between men and women. These two tests are equivalent to the gender interaction effects in the interaction approach model using MLM, and you can see that we draw the same conclusions here.

Between-Dyads Moderator

Indistinguishable Dyads

Multilevel Modeling. When using MLM two interaction terms are included—the interaction of the moderator and the actor variable and the interaction of the moderator and the partner variable—as well as a main effects of actor, partner, and the moderator:

$$Y_{ij} = b_0 + b_1X_{ij} + b_2X'_{ij} + b_3M_j + b_4X_{ij}M_j + b_5X'_{ij}M_j + e_{ij}. \quad (8)$$

The significance tests of these interactions indicate whether or not the between-dyads moderator interacts with the actor and partner effects, coefficients b_4 and b_5 , respectively. Additionally, if the moderator is continuous, for interpretability one can estimate the simple actor and partner effects for one standard deviation above and below the mean of the moderator.

Using the example dataset we ran this model using MLM; the syntax for SPSS is as follows:


```

MIXED RESPAR WITH DEPScorec_A DEPScorec_P DecTogethc
  /FIXED= DEPScorec_A DEPScorec_P DecTogethc DEPScorec_A*DecTogethc
  DEPScorec_P*DecTogethc
  /PRINT=SOLUTION TESTCOV
  /METHOD = ML
  /REPEATED=PARTNUM | SUBJECT(FAMID) COVTYPE(CSR) .

```

The two predictors (i.e., actor's depression and partner's depression) as well as the moderator (i.e., decades together) are centered. The estimates for the effects in Equation 8 are $b_0 = 3.894, p < .01$, $b_1 = -0.026, p < .01$, $b_2 = -0.028, p < .01$, $b_3 = -0.030, p = .643$, $b_4 = -0.005, p = .524$, $b_5 = -0.022, p < .01$. There are statistically significant main effects of actor depression and partner depression—for couples at the mean of decades together—and there is not statistically significant main effect of decades together. Decades together does not moderate the actor effect of depression on happiness, but it does moderate the partner effect. The negative interaction term indicates that the longer couples have been together the greater the negative impact of one's partner's depression on the actor's happiness with roles.

For ease of interpretation of this moderation effect, we can estimate the partner effect for one standard deviation above and below the mean of decades together. To do this we need to create two variables, one where the zero point for decades together represents one standard deviation above the mean and second variable where zero represents one standard deviation below the mean. The following syntax is used to complete this:

```

COMPUTE DecTogetherHIGH = DecTogethc - .74247.
COMPUTE DecTogetherLOW = DecTogethc + .74247.
EXECUTE.

```

where 0.74247 is the standard deviation of the decades together variable. Now, using these new variables in the MLM syntax shown above, the partner effect would be the partner effect for one standard deviation above or below the mean. There is a statistically significant simple effect of partner's depression on happiness at one standard deviation above the mean in decades together, b

= -0.044, $p < .01$, and the partner effect is *not* statistically significant at one standard deviation below the mean in decades together, $b = -0.011$, $p = .174$.

Structural Equation Modeling. When using SEM to estimate a model with a between-dyads moderator, the two interaction terms described above need to be included as measured variable for each member of the dyad. The equation for member 1 is

$$Y_1 = b_0 + b_1X_1 + b_2X_2 + b_3M + b_4X_1M + b_5X_2M + e_1 \quad (9)$$

and for member 2 is

$$Y_2 = b_6 + b_7X_2 + b_8X_1 + b_9M + b_{10}X_2M + b_{11}X_1M + e_2. \quad (10)$$

Various constraints need to be placed on the model to account for the indistinguishability (Olsen & Kenny, 2006). Although the test of distinguishability for this example clearly fails ($\chi^2(15) = 65.03$, $p < .001$) indicating that the dyads are distinguishable, we nonetheless use the example to illustrate our approach for this case using the 15 equality constraints (Olsen & Kenny, 2006). The equation is

$$Y = 3.894 - 0.026X - 0.028X' - 0.030M - 0.005XM - 0.022XM' + E$$

where the actor and partner effects are statistically significant ($p < .001$), as well as the moderator by partner interaction ($p = .003$). Note that these are the same estimates found when using the MLM approach. The AMOS model and equality constraints needed to estimate these effects are shown in Figure 2.

Alternatively, when the moderator is categorical, a multigroup SEM model might be estimated in which each of the separate categories would be a different group. One advantage of this approach is that we can allow for different error variances for the different categories of the moderator.

Distinguishable Dyads

If dyad members are distinguishable and there is a between-dyads moderator, then the distinguishable dyad model discussed above expands to include four additional moderation effects.

Using gender as the distinguishing variable, in this model the between-dyads variable can moderate the actor and partner effect for women and for men thus producing the four moderator effects:

- 1) The woman's actor effect moderated by the between-dyads variable,
- 2) The woman's partner effect moderated by the between-dyads variable,
- 3) The man's actor effect moderated by the between-dyads variable, and
- 4) The man's partner effect moderated by the between-dyads variable.

Typically the central question in a distinguishable dyads with a between-dyads moderator analysis is whether the actor or partner effects for the two dyad members are moderated differently by the between-dyads moderator.

Multilevel Modeling. When using MLM, the moderating effects of the between-dyads variable are included in the model as two three-way interactions of the moderating variable, the distinguishing variable and actor or partner variable. Note that all relevant two-way interactions and main effects also need to be included in this model:

$$Y_{ij} = b_{0j} + b_{1j}X_{ij} + b_{2j}X'_{ij} + b_{3j}G_{ij} + b_{4j}M_{ij} + b_{5j}X_{ij}G_{ij} + b_{6j}X'_{ij}G_{ij} + b_{7j}M_{ij}G_{ij} + b_{8j}X_{ij}M_{ij} + b_{9j}X'_{ij}M_{ij} + b_{10j}X_{ij}G_{ij}M_{ij} + b_{11j}X'_{ij}G_{ij}M_{ij} + e_{ij}. \quad (11)$$

The test of these two three-way interactions indicates whether the between-dyads moderator differentially moderates the actor effect across dyad members and differentially moderated the partner effect across dyad members. To estimate the moderation of the actor and partner effects by the between-dyads moderator separately for each dyad member the “two-intercept” model can be used. This model yields four separate tests of the moderator—the interaction of the moderator with the actor and partner variables for each of the two members. For example, Givertz, Segrin, and Hanzal (2009) were interested in if the actor and partner effects of satisfaction on commitment for husbands and wives differed across couple type (i.e., traditional, independent or separated). To answer this question

they estimated the four effects (husband-actor and partner, and wife-actor and partner) for each couple type separately. Their results seem to indicate that actor and partner effects of satisfaction on commitment differed across couple type.

Using the example dataset we estimated both the interaction approach model and the two-intercept model with MLM. The syntax for the interaction approach model was:

```
MIXED RESPAR BY GENDER WITH DEPScorec_A DEPScorec_P DecTogethc
  /FIXED= DEPScorec_A DEPScorec_P GENDER DecTogethc
          DEPScorec_A*GENDER DEPScorec_P*GENDER DecTogethc*GENDER
          DEPScorec_A*DecTogethc DEPScorec_P*DecTogethc
          DEPScorec_A*DecTogethc*GENDER DEPScorec_P*DecTogethc*GENDER
  /PRINT=SOLUTION TESTCOV
  /METHOD = ML
  /REPEATED = GENDER | SUBJECT(FAMID) COVTYPE(CSH) .
```

With this model we find again, as in the indistinguishable case, main effect of actor depression, $b_1 = -0.029$, $p < .01$, and partner depression, $b_2 = -0.027$, $p < .01$, (for couples at the mean of decades together because this variable was centered), we find again that decades together moderates the partner effect of depression on happiness (for both men and women since gender was effect coded), $b_9 = -0.019$, $p = .014$ —no other two-way interactions are significant. We also find a main effect of gender such that men are happier with their role responsibility, $b_3 = -0.151$, $p < .01$ (for actor and partner depression at the mean and decades together at the mean). Of most importance to this model there is a three-way interaction between actor's depression, decades together and gender suggesting that the moderation of the actor effect by the between dyads moderator is statistically different across genders. The aid in the interpretation of this three-way interaction the two-intercept model can be estimated which we do next.

The syntax for the two-intercept model is as follows:

```
MIXED RESPAR BY GENDER WITH DEPScorec_A DEPScorec_P DecTogethc
  /FIXED= GENDER DEPScorec_A*GENDER DEPScorec_P*GENDER DecTogethc_A*GENDER
          DEPScorec_A*DecTogethc*GENDER DEPScorec_P*DecTogethc*GENDER|NOINT
  /PRINT=SOLUTION TESTCOV
  /METHOD = ML
  /REPEATED=GENDER | SUBJECT(FAMID) COVTYPE(CSH) .
```

With this model we can see that for men the interaction effect of actor's depression and decades together is positive, $b = 0.024$, $p = .019$ and it is negative for women, $b = -0.031$, $p = .005$. This indicates that the longer the couple is together the husband's own depression stops affecting his own happiness, but the longer the couple is married the wife's own depression starts negatively affecting her own happiness even more. Within the two-intercept model, the DecTogetherHIGH and DecTogetherLOW variables can be used to see the actor effects for husbands and wives when the couple has been together one standard deviation above and below the mean. Here is the syntax:

```
MIXED RESPAR BY GENDER WITH DEPScorec_A DEPScorec_P DecTogethHIGH
  /FIXED= GENDER DEPScorec_A*GENDER DEPScorec_P*GENDER DecTogethHIGH*GENDER
DEPScorec_A*DecTogethHIGH*GENDER DEPScorec_P*DecTogethHIGH*GENDER|NOINT
  /PRINT=SOLUTION TESTCOV
  /METHOD = ML
  /REPEATED= GENDER | SUBJECT(FAMID) COVTYPE(CSH) .

MIXED RESPAR BY GENDER WITH DEPScorec_A DEPScorec_P DecTogethLOW
  /FIXED= GENDER DEPScorec_A*GENDER DEPScorec_P*GENDER DecTogethLOW*GENDER
DEPScorec_A*DecTogethLOW*GENDER DEPScorec_P*DecTogethLOW*GENDER|NOINT
  /PRINT=SOLUTION TESTCOV
  /METHOD = ML
  /REPEATED= GENDER | SUBJECT(FAMID) COVTYPE(CSH) .
```

At one decade above the mean having lived together there is no statistically significant actor effect for husbands, $b = -0.015$, $p = .134$, and a statistically significant negative actor effect for wives, $b = -0.049$, $p < .01$. At one standard deviation below the mean in decades together there is a statistically significant negative actor effect of husbands, $b = -0.050$, $p < .01$, and now *no* statistically significant actor effect for wives, $b = -0.003$, $p = .795$!

Structural Equation Modeling. For SEM, the main effect of the moderator and its interaction with X_1 and X_2 are added to the model, yielding four different moderator effects:

$$Y_1 = b_0 + b_1X_1 + b_2X_2 + b_3M + b_4X_1M + b_5X_2M + e_1 \quad (12)$$

and

$$Y_2 = b_6 + b_7X_2 + b_8X_1 + b_9M + b_{10}X_2M + b_{11}X_1M + e_2 \quad (13)$$

We note that these equations are identical with those of the indistinguishable model with a between-dyads moderator, but the indistinguishable model requires a set of equality constraints. By imposing new equality constraints on the distinguishable model, various tests of moderator variables can be obtained. For instance, to test if actor moderation is the same for both members, the path from the moderator times X_1 to Y_1 and path from the moderator times X_2 to Y_2 are set equal. Such a test is equivalent within MLM modeling of actor by moderator by distinguishing variable interaction.

In sum, with the between-dyads moderator with distinguishable dyads, there are four moderator effects: The moderator can interact with actor and with partner variables and these can be computed for each of the two members. It is important to compute and test these four effects and determine if in fact they differ across the two members. Again using the same example dataset and between dyads moderator, decades together, we estimated this model with SEM. The model shown in Figure 2 is used except there are no equality constraints. We find that the estimates are the same as in the distinguishable dyads, two-intercept approach model. With women as member 1 and men as member 2, from equations 12 and 13 above these estimates are $b_0 = 3.734, p < .01, b_1 = -0.026, p = .006, b_2 = -0.036, p < .01, b_3 = -0.001, p = .988, b_4 = -0.031, p = .005, b_5 = -0.014, p = .244, b_6 = 4.036, p < .01, b_7 = -0.033, p < .01, b_8 = -0.018, p = .021, b_9 = -0.026, p = .717, b_{10} = 0.024, p = .019, \text{ and } b_{11} = -0.023, p = .011$.

To test in the moderation of the actor and partner effects is statistically different across members we can estimate 1) a model that constrains the actor moderation paths for men and women to be equal, and 2) a second model that constrains the partner moderation paths for men and women to be equal. Comparing the fit of these models to that of the unconstrained model will test whether or not the moderation is different across gender (this test being analogous to the three-way interactions in the interaction approach model within MLM). The first model indicates that there is a significant decline in

fit when constraining the actor moderation paths to be equal, $\chi^2(1) = 12.905$, $p < .001$, and thus, as we found in the interaction approach model for MLM, the moderation of the actor effect is different between husbands and wives. There is not statistically significant decline in fit when constraining the partner moderation paths to be equal, $\chi^2(1) = 0.326$, $p = .568$, and so there is no difference in this moderation of the partner effect between husbands and wives.

Mixed Moderator

If the moderator is mixed (a variable that varies both between and within dyads), then there are actually two potential moderators of the actor and partner effect. We call one the *actor moderator* which is the person's own score on the moderator and the other the *partner moderator* which is the person's partner's score on the moderator. So for instance, in examining the effect of attachment style on relationship satisfaction, a person's own neuroticism, an actor moderator, might moderate the actor and partner effects of attachment style on satisfaction, or a person's partner's neuroticism, a partner moderator, might moderate these effects. Because both the X , attachment in this example, and the moderator are mixed variables, the researcher can decide to flip the two. For instance, make attachment style the moderator and neuroticism the X variable: A person's own attachment style can alters the actor and partner effects of neuroticism on relationship satisfaction, and his or her partner's attachment style can alters these effects. The two are statistically equivalent, but often one way may make more sense theoretically.

Indistinguishable dyads

The possible moderation effects in the APIM become much more complicated when a mixed variable is used as a moderator. In the indistinguishable case, two separate moderation variables are included in the model: The actor's moderator and the partner's moderator. This inclusion produces four interaction terms added to the model:

- 1) The moderation of the actor effect by the actor's moderator,
- 2) The moderation of the actor effect by the partner's moderator,
- 3) The moderation of the partner effect by the actor's moderator, and
- 4) The moderation of the partner effect by the partner's moderator.

Note that these four interaction terms are in addition to the four main effects that also need to be in the model. To estimate the APIM model with a mixed moderator, either MLM or SEM can be used.

Multilevel Modeling. For MLM, four two-way interaction terms are added to the model—the interaction of the actor moderator with both the actor and partner variables and the interaction of the partner moderator with both the actor and partner variables. All relevant main effects are included in this model as well:

$$Y_{ij} = b_{0j} + b_{1j}X_{ij} + b_{2j}X'_{ij} + b_{3j}M_{ij} + b_{4j}M'_{ij} + b_{5j}X_{ij}M_{ij} + b_{6j}X_{ij}M'_{ij} + b_{7j}X'_{ij}M_{ij} + b_{8j}X'_{ij}M'_{ij} + e_{ij}. \quad (14)$$

The tests of these four interactions terms determine whether the actor or partner moderators significantly change the direction or magnitude of the actor and partner effects. In the following analyses we use the same example dataset, but this time happiness with conflict resolution is the outcome variable, work family conflict is the predictor variable and stress is the moderator (a mixed moderator). The actor and partner effects of work-family conflict will be estimated and both actor's stress and partner's stress will be used as moderators. To estimate the model in Equation 14 the following syntax was used:

```
MIXED CONFLC_A WITH WKFAMAc_A WKFAMAc_P STRESSc_A STRESSc_P
  /FIXED= WKFAMAc_A WKFAMAc_P STRESSc_A STRESSc_P
  STRESSc_A*WKFAMAc_A STRESSc_P*WKFAMAc_A
  STRESSc_A*WKFAMAc_P STRESSc_P*WKFAMAc_P
  /PRINT=SOLUTION TESTCOV
  /METHOD = ML
  /REPEATED=PARTNUM | SUBJECT (FAMID) COVTYPE (CSR) .
```

where both the predictor and moderator variables have been centered for interpretability. The estimates for the model parameters are as follows: $b_0 = 3.789$ ($< .01$), $b_1 = -0.030$ (.636), $b_2 = -0.089$ (.165), b_3

$= -0.233 (< .01)$, $b_4 = -0.100 (.113)$, $b_5 = -0.048 (.440)$, $b_6 = -0.077 (.320)$, $b_7 = -0.086 (.266)$, and $b_8 = -0.123 (.047)$. We see that there is a significant main effect of the actor's moderator, stress (b_3), on happiness such that the more stress the actor is feeling the less happy he or she is with conflict resolution (note: this effect is for those who are at the mean of actor and partner's work-family conflict). Additionally, and most importantly for the goal of this model, there is a statistically significant interaction of partner's stress and partner's work-family conflict on happiness (b_8) such that more the more the partner is stressed out the greater the negative effect of the partner's work family conflict on the *actor's* happiness.

To get a better sense of this effect we can estimate the partner effect of work-family conflict at one standard deviation above and below the mean for partner's stress. As before, we first need to calculate new variables to change the meaning of zero for the partner's stress variable:

```
COMPUTE STRESS_P_HIGH = STRESSc_P - .83358.
COMPUTE STRESS_P_LOW = STRESSc_P + .83358.
EXECUTE.,
```

where .83358 is the standard deviation of the stress variable. Then we use these variables in two separate models with the following syntax:

```
MIXED CONFLC_A WITH WKFAMAc_A WKFAMAc_P STRESSc_A STRESS_P_HIGH
  /FIXED= WKFAMAc_A WKFAMAc_P STRESSc_A STRESS_P_HIGH
  STRESSc_A*WKFAMAc_A STRESS_P_HIGH*WKFAMAc_A
  STRESSc_A*WKFAMAc_P STRESS_P_HIGH*WKFAMAc_P
  /PRINT=SOLUTION TESTCOV
  /METHOD = ML
  /REPEATED=partnum | SUBJECT(FAMID) COVTYPE(CSR) .

MIXED CONFLC_A WITH WKFAMAc_A WKFAMAc_P STRESSc_A STRESS_P_LOW
  /FIXED= WKFAMAc_A WKFAMAc_P STRESSc_A STRESS_P_LOW
  STRESSc_A*WKFAMAc_A STRESS_P_LOW*WKFAMAc_A
  STRESSc_A*WKFAMAc_P STRESS_P_LOW*WKFAMAc_P
  /PRINT=SOLUTION TESTCOV
  /METHOD = ML
  /REPEATED=partnum | SUBJECT(FAMID) COVTYPE(CSR) .
```

The estimates in these models confirm the interpretation above, when one's partner's stress is high there is a statistically significant negative effect of the partner's work-family conflict on the actor's

happiness, $b = -0.192$, $p = .021$. In contrast, when the partner's stress is low there is no statistically significant effect of partner's work-family conflict on the actor's happiness, $b = 0.013$, $p = .872$.

Structural Equation Modeling. For SEM, these four interactions are added as exogenous variables in the analysis.

$$Y_1 = b_0 + b_1X_1 + b_2X_2 + b_3M_1 + b_4M_2 + b_5X_1M_1 + b_6X_1M_2 + b_7X_2M_2 + b_8X_2M_1 + e_1 \quad (15)$$

and

$$Y_2 = b_0 + b_1X_2 + b_2X_1 + b_3M_2 + b_4M_1 + b_5X_2M_2 + b_6X_2M_1 + b_7X_1M_1 + b_8X_1M_2 + e_2. \quad (16).$$

The four interactions each have two paths, and appropriate equality constraints must be made to take into account for indistinguishability. Figure 3 shows this full APIM mixed moderator SEM model with a list of the equality constraints needed for indistinguishability. Tests of the four free paths from the interaction evaluate the four different moderator effects.

Although the test of distinguishability for this example clearly fails ($\chi^2(30) = 151.166$, $p < .001$), we nonetheless use the example to illustrate our approach for this case making the 30 equality constraints necessary to make dyad members indistinguishable (Olsen & Kenny, 2006). The basic equation is

$$Y = 3.789 - 0.030X - 0.090X' - 0.232M - 0.086M' - 0.048XM - 0.077X'M \\ - 0.086XM' - 0.123X'M' + E$$

where the actor effect for the moderator is statistically significant ($p < .001$), as partner by partner interaction ($p = .046$). Note that these estimates are the same as the analysis conducted with the MLM approach above.

Distinguishable Dyads

The most complex APIM moderation model involves distinguishable dyads and mixed moderators. In the basic distinguishable dyads case, there are two actor effects and two partner effects.

If the moderating variable is mixed then one may have the actor's score on that variable as well as the partner's score on that variable moderating each of the four base distinguishable dyads effects for a total of eight interaction terms added to the model. Using gender as the distinguishable variable, the eight interaction effects are as follows:

- 1) The woman's actor effect is moderated by the woman's moderator variable,
- 2) The man to woman partner effect is moderated by the woman's moderator variable,
- 3) The woman's actor effect is moderated by the man's moderator variable,
- 4) The man to woman partner effect is moderated by the man's moderator variable,
- 5) The man's actor effect is moderated by the man's moderator variable,
- 6) The woman to man partner effect is moderated by the man's moderator variable,
- 7) The man's actor effect is moderated by the woman's moderator variable, and
- 8) The woman to man partner effect is moderated by the woman's moderator variable.

As was the case with mixed moderators and indistinguishable dyads, using only the actor's score on the mixed variable is more common in past research than using only the partner's score, or both the actor's and the partner's scores.

Multilevel Modeling. For estimation using MLM, the eight moderating effects added to the distinguishable dyad model are estimated with four two-way interactions for each of the two dyad members. This can be done with the “two-intercept” approach where separate regression equations are estimated for each member. Again, this approach will not test if the moderation is different across members. To test for these differences a model that includes four three-way interaction terms will need to be estimated—each of the four two-ways interacting with gender:

$$Y_{ij} = b_0 + b_{1j}X_{ij} + b_{2j}X'_{ij} + b_{3j}M_{ij} + b_{4j}M'_{ij} + b_{5j}G_{ij} + b_{6j}X_{ij}M_{ij} + b_{7j}X_{ij}M'_{ij} + \\ b_{8j}X'_{ij}M_{ij} + b_{9j}X'_{ij}M'_{ij} + b_{10j}X_{ij}G_{ij} + b_{11j}X'_{ij}G_{ij} + b_{12j}M_{ij}G_{ij} + b_{13j}M'_{ij}G_{ij} +$$

$$b_{14}X_{ij}M_{ij}G_{ij} + b_{15}X_{ij}M'_{ij}G_{ij} + b_{16}X'_{ij}M_{ij}G_{ij} + b_{17}X'_{ij}M'_{ij}G_{ij} + e_{ij} \quad (17)$$

The significance test for the first interaction term indicates whether the moderation of the actor effect by the actor moderator is different across dyad member while the test for the second measures if the moderation of the partner effect by the actor moderator is different across dyad member. The significance tests for the last two interaction terms indicate if the moderation of the actor and partner effects by the partner's moderator is different across dyad members.

This model was estimated using the example dataset and the following syntax:

```
MIXED CONFLC_A WITH GENDER WKFAMAC_A WKFAMAC_P STRESSC_A STRESSC_P
  /FIXED= WKFAMAC_A WKFAMAC_P STRESSC_A STRESSC_P GENDER
  STRESSC_A*WKFAMAC_A STRESSC_P*WKFAMAC_A
  STRESSC_A*WKFAMAC_P STRESSC_P*WKFAMAC_P
  WKFAMAC_A*GENDER WKFAMAC_P*GENDER
  STRESSC_A*GENDER STRESSC_P*GENDER
  STRESSC_A*WKFAMAC_A*GENDER STRESSC_P*WKFAMAC_A*GENDER
  STRESSC_A*WKFAMAC_P*GENDER STRESSC_P*WKFAMAC_P*GENDER
  /PRINT=SOLUTION TESTCOV
  /METHOD = ML
  /REPEATED= GENDER| SUBJECT(FAMID) COVTYPE(CSH) .
```

Gender is effect coded with men as -1 and women as 1. The predictor variables and moderator variables are centered for interpretability. The estimates of the parameters in equation 17 are as follows: $b_0 = 3.791, p < .01, b_1 = -0.067, p = .327, b_2 = -0.131, p = .065, b_3 = -0.220, p = .001, b_4 = -0.096, p = .136, b_5 = 0.020, p = .608, b_6 = -0.035, p = .596, b_7 = -0.018, p = .829, b_8 = -0.071, p = .404, b_9 = -0.172, p = .012, b_{10} = 0.146, p = .042, b_{11} = -0.108, p = .142, b_{12} = 0.029, p = .648, b_{13} = -0.017, p = .797, b_{14} = -0.050, p = .455, b_{15} = -0.012, p = .893, b_{16} = -0.038, p = .674, and $b_{17} = -0.102, p = .143$. Thus, because the four three-way interactions were all not statistically significant, then we can conclude that the moderation is not different across genders. As for the two-way interactions, we do see that the actor effect of work family conflict is different across genders (b_{10}) and again, the interaction of partner's work family conflict and partner's stress is significant (b_9).$

Despite the lack of moderation differences across gender, it might still be useful for pedagogical reasons to estimate the two-intercept model. The following syntax is used to estimate this model:

```
MIXED CONFLC_A BY GENDER WITH WKFAMAc_A WKFAMAc_P STRESSc_A STRESSc_P
  /FIXED= GENDER WKFAMAc_A*GENDER WKFAMAc_P*GENDER STRESSc_A*GENDER
  STRESSc_P*GENDER
  STRESSc_A*WKFAMAc_A*GENDER STRESSc_P*WKFAMAc_A*GENDER
  STRESSc_A*WKFAMAc_P*GENDER STRESSc_P*WKFAMAc_P*GENDER|NOINT
  /PRINT=SOLUTION TESTCOV
  /METHOD = ML
  /REPEATED=GENDER | SUBJECT(FAMID) COVTYPE(CSH) .
```

Now we can see if each of the four moderation effects for each dyad member is statistically different from zero. We find that the only one of these effects that is statistically different from zero is the interaction of partner's work family conflict and partner's stress for women, $b = -0.274$, $p = .022$. But again, we already know that this moderation effect is not significantly different across dyad members.

Structural Equation Modeling. For SEM, the four interactions of two moderators with actor and partner effects are added as predictors. Because dyad members are treated as distinguishable, their effects on Y_1 and Y_2 are allowed to differ, which results in a total of eight interaction effects:

$$Y_1 = b_0 + b_1X_1 + b_2X_2 + b_3M_1 + b_4M_2 + b_5X_1M_1 + b_6X_1M_2 + b_7X_2M_2 + b_8X_2M_1 + e_1 \quad (18)$$

And

$$Y_2 = b_9 + b_{10}X_2 + b_{11}X_1 + b_{12}M_2 + b_{13}M_1 + b_{14}X_2M_2 + b_{15}X_2M_1 + b_{16}X_1M_1 + b_{17}X_1M_2 + e_2 \quad (19).$$

Again, these equations are identical with those of the indistinguishable model. The same model that appears in Figure 3 is used to estimate this model except the equality constraints are not used. This is the analysis technique used in the paper and the estimates are presented there in Table 6.

To test if the moderation is significantly different across dyad members, the parameters for the two relevant interaction effects can be set equal across members and a chi-square test can be used.

Combining X and M Models

The pattern for X can also be combined with the pattern for M . For instance, the study might result in an actor-only moderator model—only the actor’s moderator is used—with a contrast model for X —the actor and partner effects are moderated to the same extent but in the opposite direction by the actor moderator. We might first select the best fitting model for the four ways of conceptualizing the moderator: actor M only, partner M only, couple M , and contrast M . If dyad members are distinguishable, this would be done separately for each member. Then one would estimate the four models for the effect of X : actor X only, partner X only, couple X , and contrast X . Thus, when we have a mixed moderator finding a moderation pattern involves transforming the two X variables (actor and partner) and the two M variables (actor and partner) into a single variable, the final model can then be described in terms of X and M . Table 1 contains all of the possible combinations of X and M models for the indistinguishable dyads case. If the dyads are distinguishable each type of dyad member can have a different pattern.

APIM Moderation using k : Relating Main Effect to Moderator Effects

So far, we have been interpreting the moderator effects without explicitly comparing them to the main effects of X and M . Moreover, our strategy for creating patterns has been to transform the two X s and M s when M is mixed into a single variable when forming the interaction. We may want to use that same transformation for the main effects. One way to accomplish this is to use a parameter that Kenny and Ledermann (2010) have proposed: k is the ratio of the partner effect to the actor effect. When k is 1 we have couple level model, k equal to -1 is contrast model, and k equal to zero is actor-only model. In between models are possible: k might be less than 1 but greater than 0 (e.g., 0.6).

For moderation, k can be used to place constraints on the interaction coefficients. For an indistinguishable model with between-dyads moderator, we can define k as the ratio of the partner effect to the actor and k_M as the ratio of the interaction effect of XM to the effect of $X'M$. If we assume

that $k = k_M$ which we describe as the *common k model*, the variable is X is transformed into $X + kX'$.

That is, the moderator changes both the actor and partner effects, but the ratio of partner to actor effects does not change as the moderator changes. Thus, in some sense, k “explains” the moderation. The constraint that $k = k_M$ can be tested statistically by comparing the fit of two models, one where the constraint is made and the other where it is not.

We estimated the common k model for earlier described between-dyads moderator of Decades Together. This model is presented in Figure 4. For Husbands, the fit testing a common k model was poor ($\chi^2(1) = 9.68, p = .002$), but it was good for wives ($\chi^2(1) = 1.50, p = .221$) and the value of common value of k for wives was 1.05 and its 95% bias-corrected bootstrapped confidence interval (based on 5000 bootstrap samples) ranges from 0.31 to 3.34. Thus, we conclude for wives, k is not moderated by Decades Together, but for husbands it is. The parameter k increases for husbands the longer that the couple has been together.

If M is mixed and dyad members are indistinguishable, the multilevel equation is as follows:

$$Y = aX + pX' + mM + nM' + qXM + zX'M + fXM' + gX'M' + E \quad (20)$$

We define $k_X = p/a$ and $k_M = n/m$ and the common k constraints would be $z = k_Xq = k_Mf = k_Xk_Mg$. We can rewrite, Equation 6 for the common k model as

$$Y = a(X + k_XX') + m(M + k_MM') + q(X + k_XX')(M + k_MM') + E \quad (21)$$

Note in Equation x4 the interaction variable is the product the two “main effects” scaled by the two k parameters and when we multiply out $q(X + k_XX')(M + k_MM')$ we obtain $qXM + qk_XX'M + qk_MM' + qk_Xk_MM'$. Note that the model has three constraints in that the four interaction effects have just one unique parameter, q .

We estimated the common k model for the mixed moderator (shown in Figure 5), we obtained a good fit ($\chi^2(12) = 13.57, p = .329$) overall and the test of the three constraints in the common k model

were not statistically significant ($\chi^2(3) = 3.13, p = .372$), indicating that the constraints do hold.

However, we find that $k_X = 3.19$ with huge standard errors. Note that in the saturated model (see Table 6 of the paper), the main effects for actor are not statistically significant and of opposite sign. The standardized actor effect in the indistinguishable case is only $-.03$, which is well below the minimum value for computing k of 0.10 recommended by Kenny and Lederman (2010). However, $k_M = 0.54$ and is significantly greater than zero, but less than one. We do note that when different values of k are estimated for the main effects and interaction, we find $k_X = 1.71$ and $k_M = 1.49$ for moderation.

For the APIM, we often find values of k that are less than 1, i.e., with actor effect being larger than partner effects. However, a case can be made that actor effects are inflated because both X and Y are from the same respondent and this inflation leads to artificially low values of k . A case can be made that z/q (and g/f for a mixed moderator) might provide a more “honest” albeit more unstable estimate of k .

References

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Table 1

Combined X and M Pattern Models for a Mixed Moderator with Indistinguishable Dyads

Moderation	X Model	Actor Only	Actor Only	Actor Only	Actor Only	Partner Only	Partner Only	Partner Only	Partner Only
Effect	M Model	Actor Only	Partner Only	Couple	Contrast	Actor Only	Partner Only	Couple	Contrast
Actor X with Actor M		a	0	a	a	0	0	0	0
Partner X with Actor M		0	0	0	0	a	0	a	a
Actor X with Partner M		0	a	a	-a	0	0	0	0
Partner X with Partner M		0	0	0	0	0	a	a	-a
Moderation	X Model	Couple	Couple	Couple	Couple	Contrast	Contrast	Contrast	Contrast
Effect	M Model	Actor Only	Partner Only	Couple	Contrast	Actor Only	Partner Only	Couple	Contrast
Actor X with Actor M		a	0	a	a	a	0	-a	a
Partner X with Actor M		a	0	a	a	-a	0	a	-a
Actor X with Partner M		0	a	a	-a	0	a	-a	-a
Partner X with Partner M		0	a	a	-a	0	-a	a	a

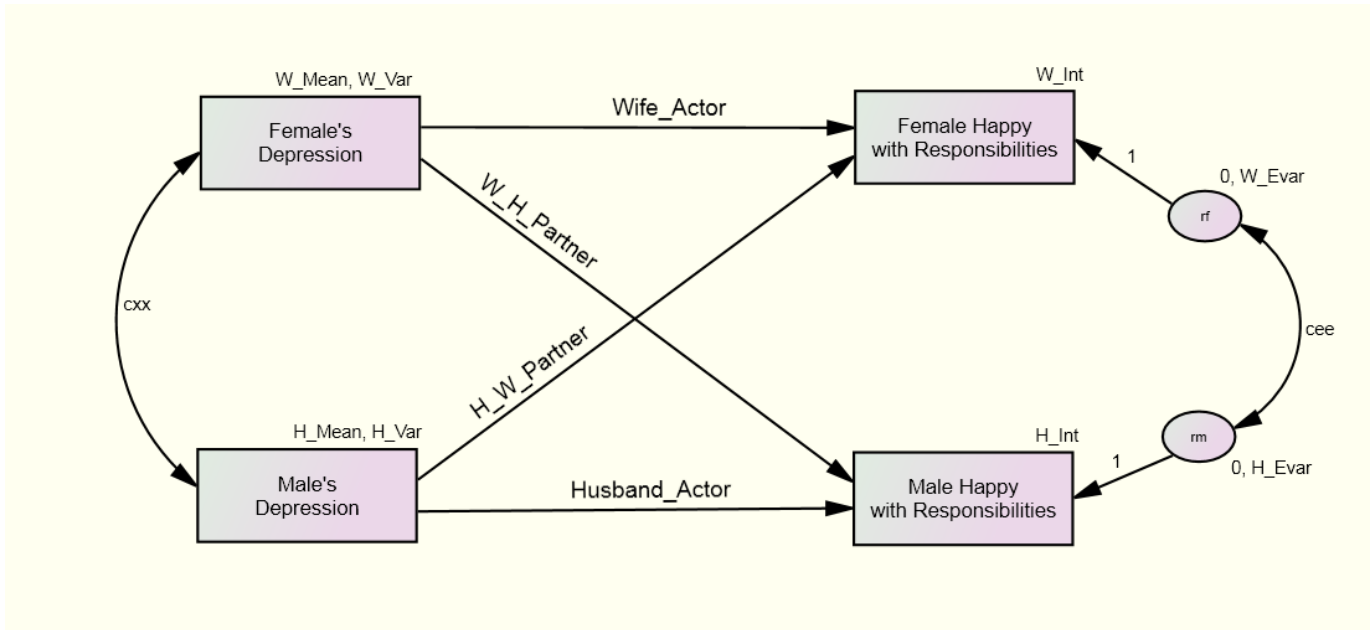


Figure 1. Indistinguishable Dyads. The following constraints are made on this model: 1) $H_Int = W_Int$, 2) $H_Mean = W_Mean$, 3) $H_Var = W_Var$, 4) $H_Evar = W_Evar$, 5) $Husband_Actor = Wife_Actor$, 6) $H_W_Partner = W_H_Partner$

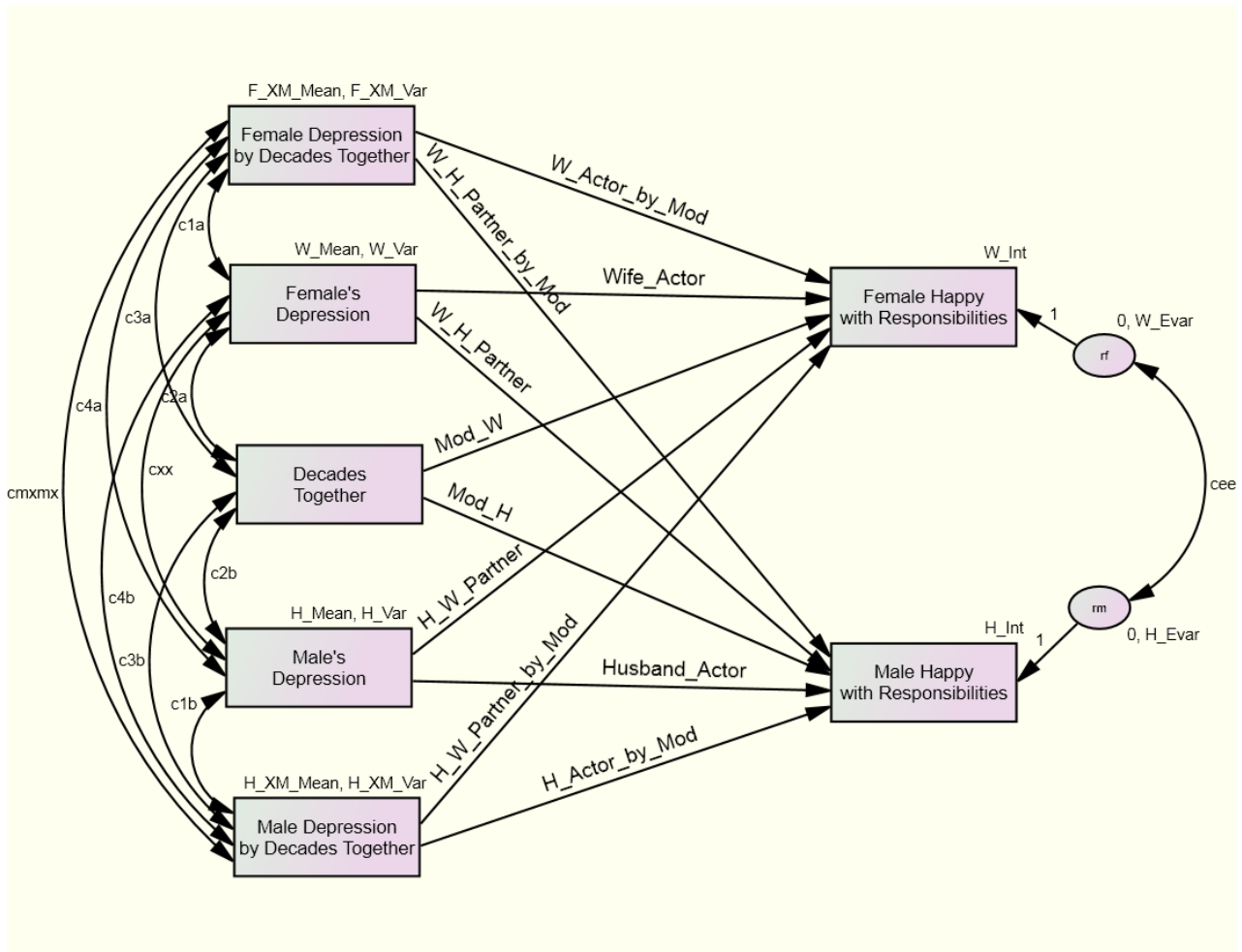


Figure 2. SEM Model with a Between Dyads Moderator. For indistinguishable dyads the following equality constraints are needed: 1) $H_Int = W_Int$, 2) $H_Mean = W_Mean$, 3) $H_XM_Mean = F_XM_Mean$, 4) $H_Var = W_Var$, 5) $F_XM_Var = H_XM_Var$, 6) $H_Evar = W_Evar$, 7) $Husband_Actor = Wife_Actor$, 8) $H_W_Partner = W_H_Partner$, 9) $H_W_Partner_by_Mod = W_H_Partner_by_Mod$, 10) $H_Actor_by_Mod = W_Actor_by_Mod$, 11) $Mod_H = Mod_W$, 12) $c1a = c1b$, 13) $c2a = c2b$, 14) $c3a = c3b$, 15) $c4a = c4b$

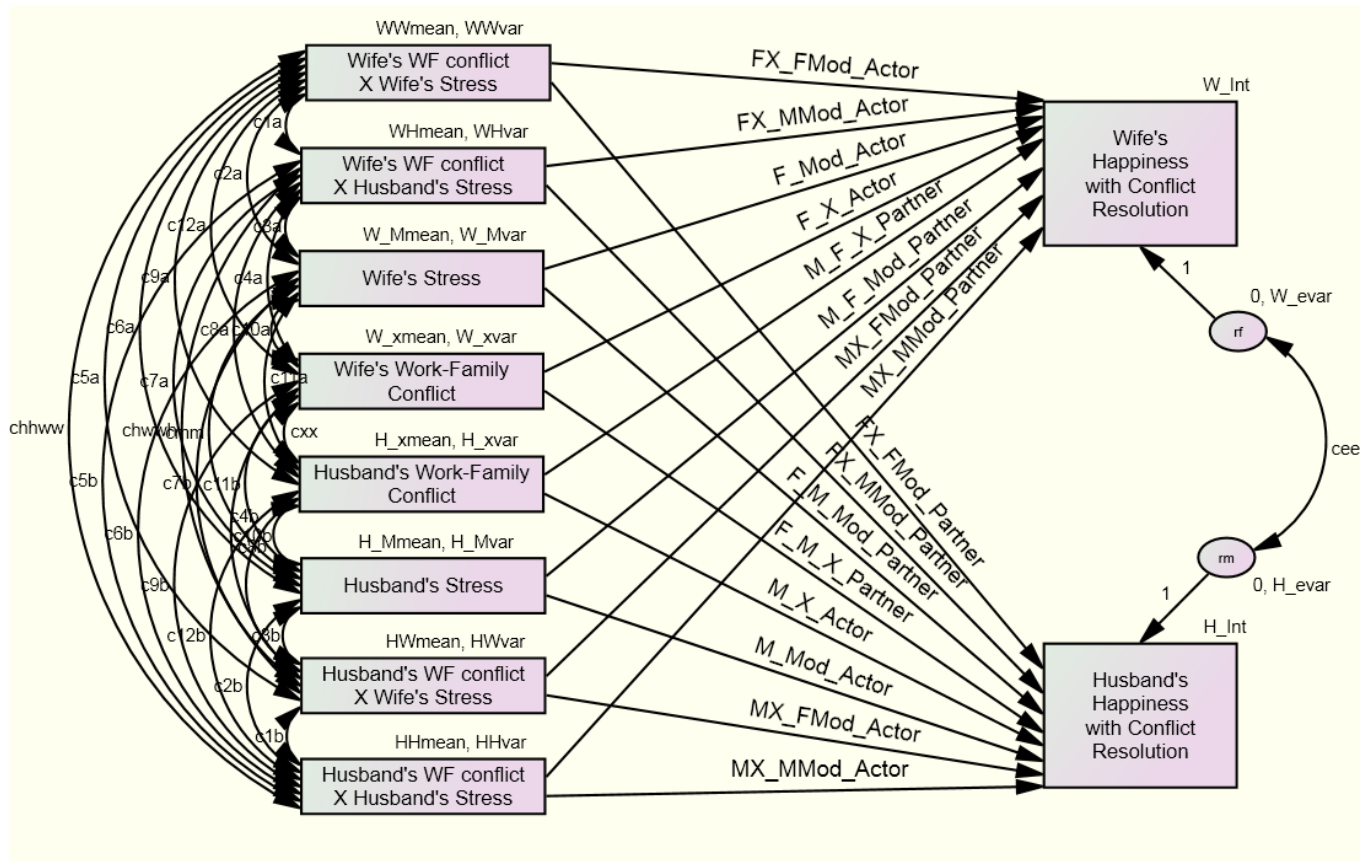


Figure 3. SEM Model with a Mixed Moderator. For indistinguishable dyads the following equality constraints are needed: 1) $H_xmean = W_xmean$, 2) $H_Mmean = W_Mmean$, 3) $HHmean = WWmean$, 4) $HWmean = WHmean$, 5) $W_Int = H_Int$, 6) $H_evar = W_evar$, 7) $H_Mvar = W_Mvar$, 8) $HHvar = WWvar$ 9) $WHvar = HWvar$, 10) $H_xvar = W_xvar$, 11) $F_X_Actor = M_X_Actor$, 12) $F_M_X_Partner = M_F_X_Partner$, 13) $F_Mod_Actor = M_Mod_Actor$, 14) $F_M_Mod_Partner = M_F_Mod_Partner$, 15) $FX_FMod_Actor = MX_MMod_Actor$, 16) $FX_FMod_Partner = MX_MMod_Partner$, 17) $FX_MMod_Actor = MX_FMod_Actor$, 18) $FX_MMod_Partner = MX_FMod_Partner$, 19) $c1a = c1b$, 20) $c2a = c2b$, 21) $c3a = c3b$, 22) $c4a = c4b$, 23) $c5a = c5b$, 24) $c6a = c6b$, 25) $c7a = c7b$, 26) $c8a = c8b$, 27) $c9a = c9b$, 28) $c10a = c10b$, 29) $c11a = c11b$, and 30) $c12a = c12b$

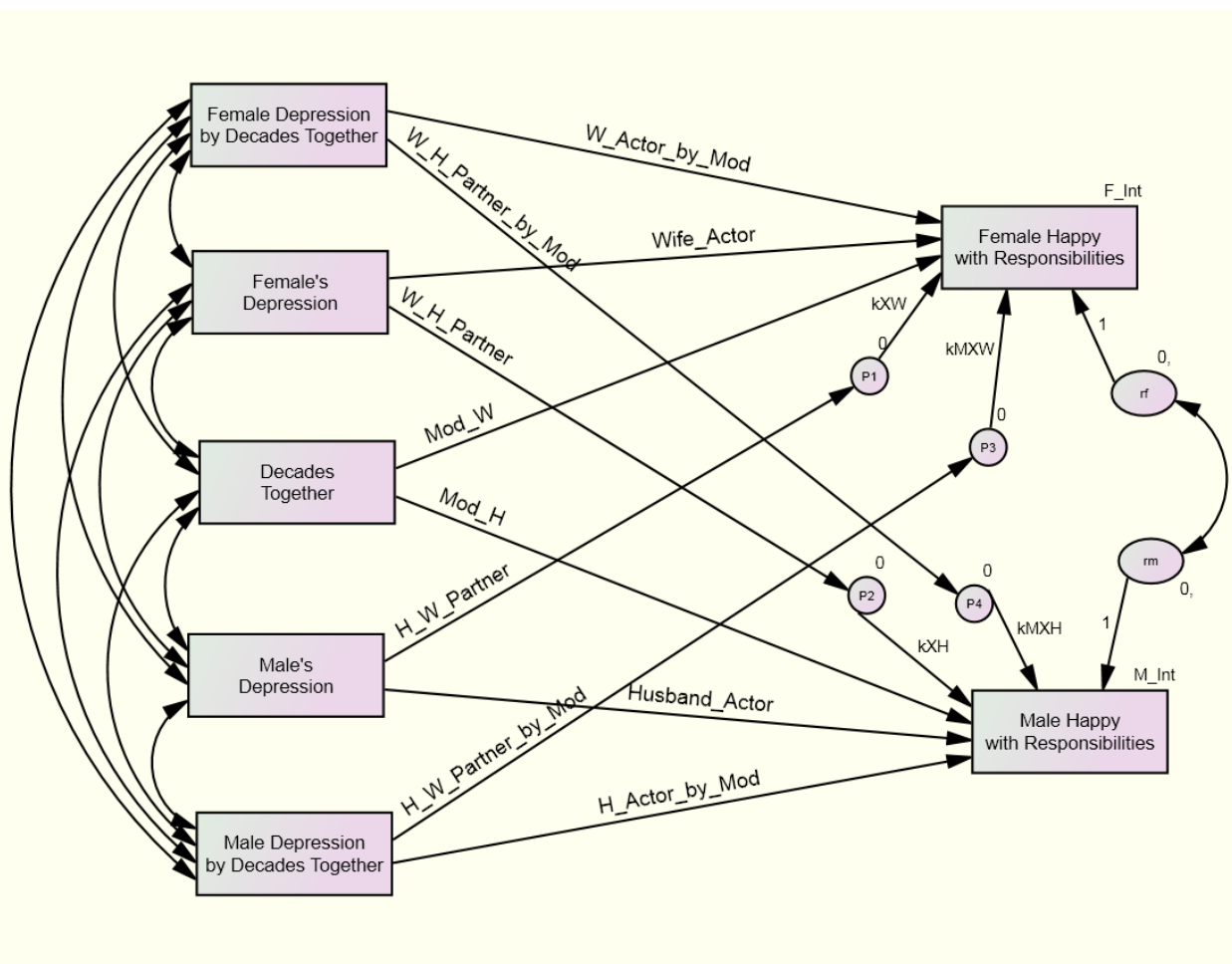


Figure 4. Using SEM to assess Moderation of the Parameter k by a Between Dyads Moderator

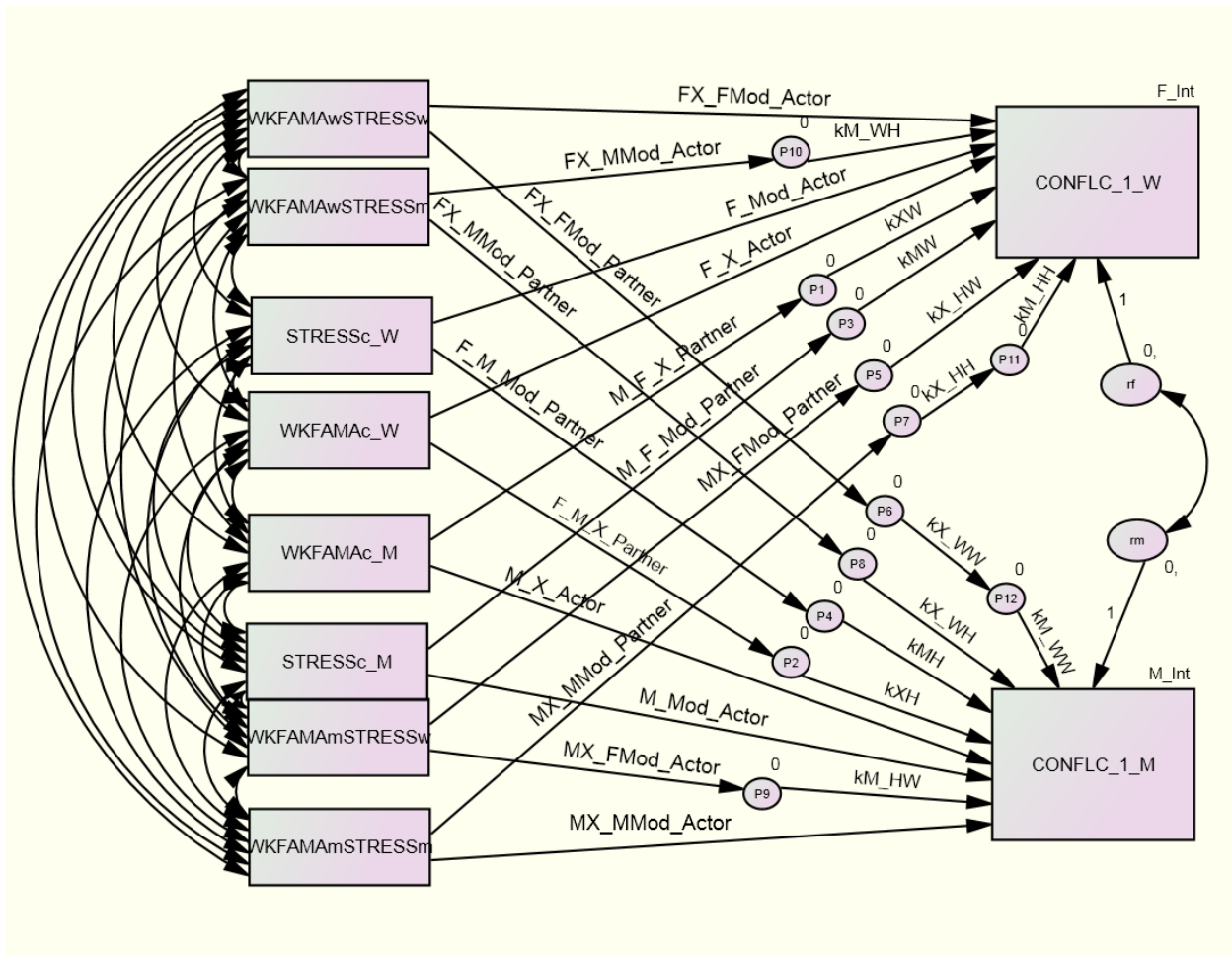


Figure 5. Using SEM to assess Moderation of the Parameter k by a Mixed Moderator